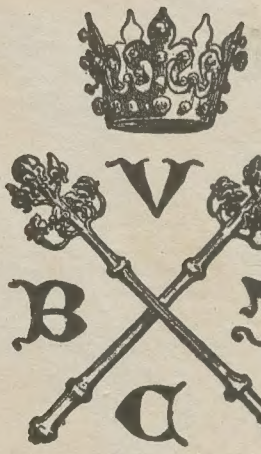

10.

CONCLUSIO CIRCULI QUADRATURÆ NOVISSIMÆ ET BREVISSIMÆ.

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§. 22. SCholion. Continuatione Quadraturæ noviſſimæ & breviſſimæ *jam impreſſa, ſed nondum publicata, contigit mihi legere manuſcriptum quoddam, cujus Autor Anonymus aſſerit, ſummam* $\frac{o+m}{mo}$ (§. 3. 13.) *innumeris modis poſſe reſolvi in 2 partes, adeoque non eſſe neceſſe, ut unus factor ſit denominator unius partis, alter factor denominator alterius, & unitas numerator utriusque partis. Falsiſſimum eſſet, inquit, ſi diceretur, ſummam e. gr.* $\frac{36}{288} = \frac{24+12}{12\times 24}$ *non poſſe conſtare ex aliis partibus, quam ex* $\frac{1}{24}$ *&* $\frac{1}{12}$: *Conſtat enim etiam ex partibus* $\frac{1}{40}$ *&* $\frac{1}{10}$ (*quoniam* $\frac{1}{40}+\frac{1}{10} = \frac{10}{400}+\frac{40}{400} = \frac{10}{400} = \frac{1}{8} = \frac{36}{288}$). *Poſſet aliorum exemplorum magna copia addi. Ad quod cum venia Cl.* Anonymi *ita reſpondeo: Partes* $\frac{1}{40}$ *&* $\frac{1}{10}$ *conſtituunt quidem ſummam* $\frac{10}{400}$ æqualem $\frac{36}{288}$, *quia utrobique numerator habet ad denominatorem ſuum eandem rationem, nempe ut* 1 : 8; *ſed nunquam poſſunt conſtituere ſummam* eandem $\frac{36}{288}$. *Quæ ſunt eadem, ſunt etiam æqualia; ſed non vice verſa, quæ ſunt æqualia, ſunt etiam eadem. Nam etſi Rhomboides cum quadrato eandem baſin & æqualem altitudinem habens, ſit huic æqualis; abſurdum tamen eſſet inde concludere, quadratum eſſe Rhomboidem, vel vice verſa. Quodlibet par fractionum ſequentium* $\frac{1}{2}$ *&* $1\frac{6}{24}$; $\frac{1}{3}$ *&* $1\frac{8}{24}$; $\frac{1}{4}$ *&* $1\frac{9}{24}$; $\frac{1}{5}$ *&* $1\frac{10}{24}$; $2\frac{4}{12}$ *&* $\frac{1}{3}$; $2\frac{6}{12}$ *&* $\frac{1}{4}$; $2\frac{8}{12}$ *&* $\frac{1}{6}$; $2\frac{9}{24}$ *&* $\frac{1}{8}$ *efficit quidem eandem ſummam, nempe* $\frac{36}{288}$: *nam fractiones 1mi paris ſunt* $= \frac{6}{288}+\frac{30}{288} = \frac{36}{288}$, *2di paris* $= \frac{4}{288}+\frac{32}{288}$; *3tii* $\frac{3}{288}+\frac{33}{288}$; *4ti* $\frac{2}{288}+\frac{34}{288}$; *5ti* $\frac{28}{288}+\frac{8}{288}$; *6ti* $\frac{30}{288}+\frac{6}{288}$; *7mi* $\frac{32}{288}+\frac{4}{288}$; *8vi* $\frac{33}{288}+\frac{3}{288}$: *attamen quoniam quævis fractionum priorum habet duplicem denominatorem; palam eſt, quamlibet eſſe fractionem fractionis. Sic 1ma eſt* $\frac{1}{2}$ *de* $\frac{1}{24} = \frac{1}{48}$, *2da* $\frac{30}{24}$ *de* $\frac{1}{12} = \frac{30}{288}$, *3tia* $\frac{1}{3}$ *de* $\frac{1}{24} = \frac{1}{72}$, *4ta* $\frac{32}{24}$ *de* $\frac{1}{12} = \frac{32}{288}$, *5ta* $\frac{1}{4}$ *de* $\frac{1}{24} = \frac{1}{96}$, *6ta* $\frac{34}{24}$ *de* $\frac{1}{12} = \frac{33}{288}$, *7ma* $\frac{1}{6}$ *de* $\frac{1}{24} = \frac{1}{144}$, *8va* $\frac{34}{24}$ *de* $\frac{1}{12} = \frac{34}{288}$, *& ita porro. Jam cum numerator ſummæ per conditionem problematis (§. 3. 13.) debeat eſſe aggregatum ex utroque factore denominatoris; erit ſumma 1mi paris* $\frac{288+48}{48\times 288}$, *2di* $\frac{288+72}{72\times 288}$, *3tii* $\frac{288+96}{96\times 288}$, *4ti* $\frac{288+144}{144\times 288}$, *& ita porro, conſequenter ubique alia; ex quo manifeſtum eſt, ſummam* $\frac{36}{288}$ *innumeris modis quidem eſſe reſolubilem in partes duas; ſed nullas alias eam poſſe efficere talem, ut ejus numerator ſit aggregatum*

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ex utroque factore denominatoris, nisi $\frac{1}{24}$ *&* $\frac{1}{12}$. *Quod ut evidentius appareat, investigemus lunulam respondentem diametri quadrato* $\sqsupset$ 1. *Sit itaque hujus ratio ad illam excessiva, ut* $4 : 1\frac{1}{6} \sqsupset 24 : 7$, *& defectiva ut* $4 : \frac{2}{3} \sqsupset 12 : 2$: *erunt lunulæ per utramque inventæ* $\frac{7}{24}$ *&* $\frac{2}{12} \sqsupset \frac{84}{288}$ *&* $\frac{48}{288}$, *quæ ex se invicem ablatæ, relinquunt summam excessus & defectus* $\frac{36}{288}$, *cujus numerator est aggregatum ex utroque factore* d † a $\sqsupset$ 12 † 24 *denominatoris* ad $\sqsupset$ 24. 12 $\sqsupset$ 288. *Quoniam igitur hæc summa est conflata ex excessu & defectu* (§. 2.), *& per demonstrata nullæ aliæ partes sunt assignabiles, quæ eam ita constituant, ut ejus numerator sit aggregatum ex utroque factore denominatoris nisi* $\frac{1}{24}$ *&* $\frac{1}{12}$; *debet harum altera necessario esse excessus, altera defectus. Jam cum lunula excess:* $\frac{7}{24}$ *sit aggregatum ex vera & excessu; necesse est, ut ejus excessus sit vel* $\frac{1}{12}$, *vel* $\frac{1}{24}$; *sed* $\frac{1}{12}$ *est diversæ,* $\frac{1}{24}$ *autem ejusdem cum ea denominationis: ergo* $\frac{1}{24}$ *utpote ejus pars homogenea debet necessario esse excessus, & per consequens* $\frac{1}{12}$ *defectus: nam* $\frac{7}{24} - \frac{1}{24} \sqsupset \frac{6}{24} \sqsupset \frac{1}{4}$; *vel* $\frac{2}{12} + \frac{1}{12} \sqsupset \frac{3}{12} \sqsupset \frac{1}{4}$ *est utique lunula vera; quæ cum tam per rationes* $4 : 1\frac{1}{5}$ *&* $4 : \frac{3}{4}$; $4 : 1\frac{1}{9}$ *&* $4 : \frac{9}{10}$; $4 : 1\frac{1}{12}$ *&* $4 : \frac{10}{11}$, *quàm per innumeras alias exactissimè determinetur; evidens est, problema IV demonstrans 2dam partem Theorematis §. 3tii, esse verò verius: consequenter problema III huic superstructum firmissimo inniti fundamento. Jam cum quantitates incognitæ proportionaliter crescentes (veluti diametri cum peripheriis; quadrata diametrorum cum lunulis, circulis & segmentis; cubi diametrorum cum sphæris; quadrata* linearum constantium *cum sectionibus conicis, & quæ alia ejus generis) eadem methodo queant perfectissime determinari; dubitare non licet de usu amplissimo hujus solutionis. Me non monente autem facile intelligetur, nullatenus determinari posse e. gr. fractionem inter* $\frac{1}{2}$ *&* $\frac{1}{3}$ *cadentem, quam aliquis in mente habeat, quia limites* $\frac{1}{2}$ *&* $\frac{1}{3}$ *per nullam proportionem naturæ quæstionis conformem possunt investigari. Interim ne quid perfectioni hujus opusculi deesse videatur; lubet adhuc adjicere sequentia.*

THEOREMA II.

§. 23. *Circuli* Ludolphinus, Metianus *&* Archimedeus *peccant in excessu:* 1mus $\frac{120}{32000}$, 2dus $\frac{60}{14464}$, 3tius $\frac{2}{448}$ *quadrati diametri.*

Demonstratio. Ratio *Ludolphina* quadrati diametri ad circulum 1000 : 785 multiplicata per 32 prodit $\sqsupset$ 32000 : 25120; assumtis deinde diametri quadrato $\sqsupset$ 1, & ratione defectiva 32 : 24, prodeunt circuli $\frac{25120}{32000}$ & $\frac{24}{32} \sqsupset \frac{803840}{1024000}$ & $\frac{768000}{1024000}$, quorum differentia, h. e. summa excessus & defectus (§. 2.) est $\frac{35840}{1024000}$, cujus numerator est $\sqsupset$ 120 *d* † *a* $\sqsupset$ 3840 † 32000. Ergo excessus est $\frac{120}{32000}$ & defectus $\frac{1}{32}$, consequenter circulus verus $\frac{25120}{32000} - \frac{120}{32000} \sqsupset \frac{25000}{32000} \sqsupset \frac{25}{32}$; vel $\frac{24}{32} + \frac{1}{32} \sqsupset \frac{25}{32}$. Ratio *Metiana* 452 : 355 multiplicata per 32, producit $\sqsupset$ 14464 : 11360, quæ cum defectiva 32 : 24 prodit circulos

culos $\frac{11360}{14434}$ & $\frac{24}{32} = \frac{363520}{462848}$ & $\frac{347136}{462848}$, quorum differentia seu summa excess. & defect. est $\frac{16384}{462848}$, cujus numerator est $= 60d+a = 1920+14464$: unde (per §. 16.) excessus est $\frac{60}{14434}$ & defectus $\frac{1}{32}$: consequenter circulus verus $\frac{11360}{14434} - \frac{60}{14434} = \frac{11300}{14434}$, & reductus per 452 ad terminos minimos $= \frac{25}{32}$. Ratio *Archimedea* 14: 11 multiplicata per 32, manifestat $= 448 : 352$, quæ cum defectiva prodit circulos $\frac{352}{448}$ & $\frac{24}{32} = \frac{11264}{14336}$ & $\frac{10752}{14336}$, qui ex se invicem ablati, relinquunt summam excessus & defectus $\frac{512}{14336}$, cujus numerator est $= 2d+a = 64+448$. Quare excessus est $\frac{2}{448}$, consequenter circulus verus $\frac{352}{448} - \frac{2}{448} = \frac{350}{448}$ & reductus per 14 ad terminos minimos $= \frac{25}{32}$.

§. 24. Scholion. *Ut usus amplissimus problematis V magis magisque pateſiat, sit quadratum diametri* $= 1$, *ratio ejus ad circulum excessiva ut* 32: 26, *& defectiva, ut* 32: $24\frac{1}{7} = 96 : 73$: *erunt circuli per utramque reperti* $\frac{26}{32}$ *&* $\frac{73}{96} = \frac{2496}{3072}$ *&* $\frac{2336}{3072}$, *quorum differentia, seu summa excessus & defectus, est* $\frac{160}{3072} = d+2a = 96+64$, *adeoque excessus est* $\frac{1}{32}$ *& defectus* $\frac{2}{96}$, *consequenter circulus verus* $\frac{26}{32} - \frac{1}{32} = \frac{25}{32}$, *vel* $\frac{73}{96} + \frac{2}{96} = \frac{75}{96} = \frac{25}{32}$. *Sit deinde ratio excessiva* $32 : 25\frac{4}{5} = 160 : 129$, *& defectiva* 32: 24: *erunt circuli* $\frac{129}{160}$ *&* $\frac{24}{32} = \frac{4128}{5120}$ *&* $\frac{3840}{5120}$, *qui ex se ablati, relinquunt summam excessus & defectus* $\frac{288}{5120}$, *cujus numerator est* $= 4d+a = 128+160$: *unde excessus est* $\frac{4}{160}$ *& defectus* $\frac{1}{32}$: *consequenter circulus verus* $\frac{129}{160} - \frac{4}{160} = \frac{125}{160} = \frac{25}{32}$; *vel* $\frac{24}{32} + \frac{1}{32} = \frac{25}{32}$. *Sit denique ratio excessiva* $32 : 25\frac{3}{4} = 128 : 103$, *& defectiva* $32 : 24\frac{3}{7} = 224 : 170$: *erunt circuli* $\frac{103}{128}$ *&* $\frac{170}{224} = \frac{23072}{28672}$ *&* $\frac{21760}{28672}$, *quorum differentia prodit summam excessus & defectus* $= \frac{1312}{28672}$, *cujus numerator est* $= 3d+5a = 672+640$. *Ergo excessus est* $\frac{3}{128}$ *& defectus* $\frac{5}{224}$: *consequenter circulus verus* $\frac{103}{128} - \frac{3}{128} = \frac{100}{128} = \frac{25}{32}$; *vel* $\frac{170}{224} + \frac{5}{224} = \frac{175}{225} = \frac{25}{32}$.

§. 25. *Corollarium.* Est ergo circulus ad quadratum diametri ut $\frac{25}{32}$: $1 = 25 : 32$, consequenter diameter ad peripheriam, ut 8: 25 (§. 8.).

PROBLEMA VI.

§. 26. *Circulo par quadratum componere.*

Resolutio. 1mo. Circulus per 2 diametros secantes se ad angulos rectos, dividatur in 4 quadrantes. 2. Utraque diameter producatur 8va parte sui. 3. Ab extremitate unius diametri productæ, ad extremitatem alterius prolongatæ ducatur hypothenusa, quæ erit latus quadrati æqualis circulo.

Demonstratio. Quilibet Cathetus continet per constructionem 5 partes diametri: ergo quadrata 2 Cathetorum sunt $= 25+25 = 50$, consequenter quadratum hypothenusæ est etiam $= 50$. Atqui circulus, cujus diameter est 8 partium, est quoque $= 50$ (§. 8.). Ergo quadratum, cujus latus est dicta hypothenusa, est $=$ circulo.

§. 27. *Corollarium.* Quod si ex circuli area data ($12\frac{1}{2}$) inveniendum sit quadratum ei æquale, extrahatur ex dimidia area

($6\frac{1}{4} = \frac{25}{4}$)

($6\frac{3}{4} = \frac{45}{4}$) radix quadrata ($\frac{5}{2}$); deinde jungantur 2 lineæ huic radici æquales ad angulum rectum, & ducatur hypothenusa, quæ erit latus quadrati æqualis circulo, cujus area fuit data: nam quoniam quadrata cathetorum ($\frac{25}{4} + \frac{25}{4} = \frac{50}{4} = 12\frac{1}{2}$) efficiunt aream datam circuli, & quadratum hypothenusæ inventæ est = his quadratis cathetorum; palam est, idem quoque esse = circulo, cujus area fuit data.

THEOREMA III.

§. 28. *Sphæræ* Ludolphina, Metiana *&* Archimedea *peccant in excessu: 1ma* $\frac{36}{14400}$, *2da* $\frac{90}{12544}$, *3tia* $\frac{3}{1008}$ *cubi diametri.*

Demonstratio. Ratio *Ludolphina* cubi diametri ad sphæram 300: 157 multiplicata per 48, producit æqualem 14400: 7536; assumtis deinde cubo diam: = 1, & ratione defectiva 48: 24, prodeunt sphæræ $\frac{7536}{14400}$ & $\frac{24}{48} = \frac{3617728}{6912000}$ & $\frac{3457600}{6912000}$, quarum differentia prodit summam excessus & defectus $\frac{159728}{6912000}$, cujus numerator est = $36d+a = 1728 + 14400$: ergo excessus est $\frac{36}{14400}$ & defectus $\frac{1}{48}$ (§. 16.), consequenter sphæra vera $\frac{7500}{14400} = \frac{25}{48}$. Ratio *Metiana* 678: 355 multiplicata per 48, prodit = 32544: 17040, quæ cum defectiva producit sphæras $\frac{17040}{12544}$ & $\frac{24}{48} = \frac{817920}{1505472}$ & $\frac{761056}{1505472}$, quarum differentia manifestat summam excessus & defectus = $\frac{368864}{1505472}$, cujus numerator est = $90d+a = 4320 + 32544$: unde excessus est $\frac{90}{12544}$: consequenter sphæra vera $\frac{16750}{12544}$, & reducta per 678 ad terminos minimos = $\frac{25}{48}$. Ratio *Archimedea* 21: 11 multiplicata per 48 dat = 1008: 528, quæ cum defectiva prodit sphæras $\frac{528}{1008}$ & $\frac{24}{48} = \frac{25344}{48384}$ & $\frac{24192}{48384}$, quarum differentia sistit summam excessus & defectus = $\frac{1152}{48384}$, cujus numerator est = $3d+a = 144 + 1008$. Ergo excessus est $\frac{3}{1008}$: consequenter sphæra vera $\frac{525}{1008}$ & reducta per 21 ad terminos minimos = $\frac{25}{48}$.

§. 29. Scholion. *Numerator summæ excessus & defectus sphærarum, quarum cubus diam.* = 1, *inventæ per rationes* 48: $25\frac{2}{3}$ *&* 48: $24\frac{2}{3}$, *est* = $2d+3a$; *repertæ per rationes* 48: 26 *&* 48: $24\frac{1}{4}$, *est* = $d+3a$; *& investigatæ per rationes* 48: $25\frac{1}{8}$ *&* 48: 24, *est* = $5d+a$: *unde per* (§. 16.) *illico determinatur excessus & defectus, & ope utriusque sphæra vera, quæ ubique prodit* = $\frac{25}{48}$.

§. 30. *Corollarium.* Est ergo sphæra ad cubum diametri, ut $\frac{25}{48}$: 1 = 25: 48: consequenter diameter ad peripheriam, ut 8: 25. (§. 8.).